Scheduling and Graph Coloring

by L. Charles (Chuck) Biehl

Consider a school that requires its staff members to serve on various committees which all hold regular meetings. Trying to schedule the minimum number of meetings so that no one has to be in two places at the same time can be a difficult task, especially if the scheduling is done by trial-and-error. Here is a situation in which using a graph model can greatly reduce the difficulty of the problem.

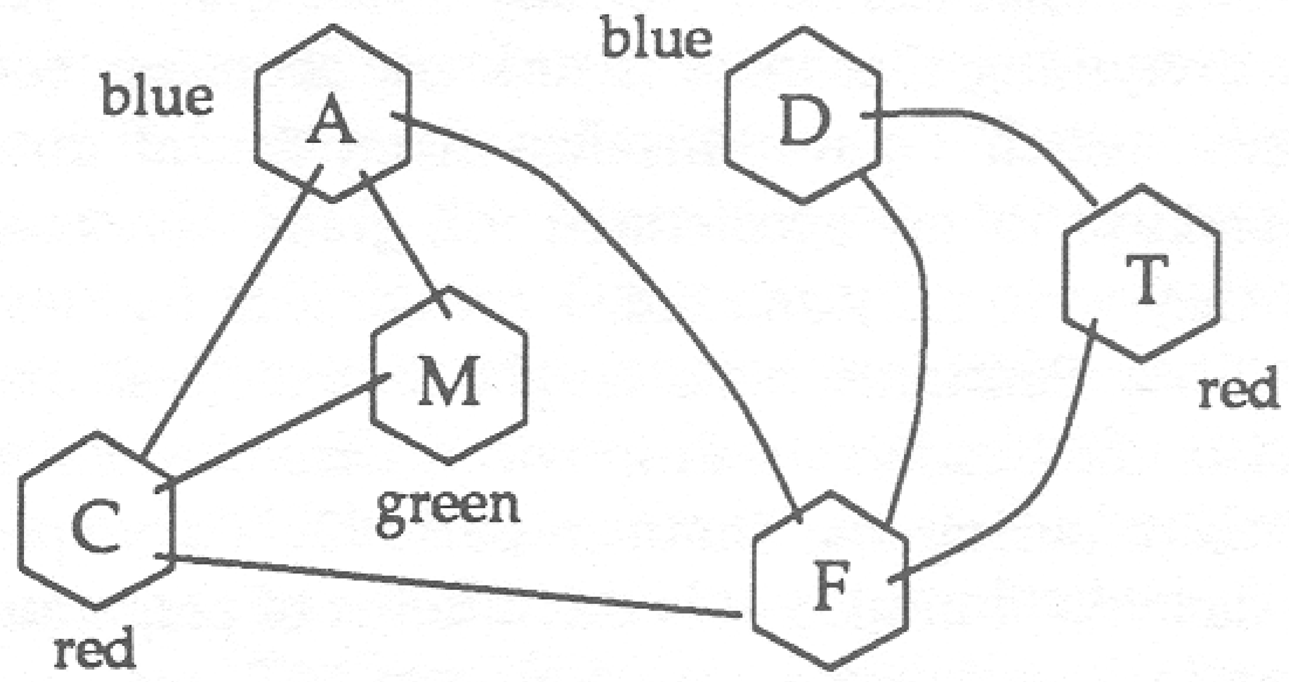
Recall that a graph is a set of points, called vertices, connected by a set of lines, called edges. In the case of a scheduling problem such as the one above, the vertices represent the committees, and the edges connect those committees which cannot meet at the same time because of a membership conflict. Such a graph is called a conflict graph.

The following is a sample problem whose solution is given below. The committees and their respective memberships are:

(C)urriculum: Davis, Franks, Grover
(D)iscipline: Bennett, Edwards, Hill
(T)extbooks: Bennett, Edwards, Isaacs
(A)ssessment: Alamos, Chavez, Davis
(F)acilities: Alamos, Edwards, Franks
(M)anagement: Chavez, Grover, Johnson

The conflict graph is shown in Figure 1. It is clear from the graph that there have to be at least three different meeting times; for example, committees M, A, and C all have conflicts with one another, as do committees F, C, and A and F, D, and T.

So where does coloring come in? Suppose that the vertices of the conflict graph are colored so that vertices joined by an edge have different colors. Then a set of vertices which are the same color represent committees that can meet at the same time. Notice that it is possible to color C and T red, D and A blue, and M and F green. This means that a possible solution to the problem is to let the first meeting time be Curriculum and Textbooks, the second meeting time Discipline and Assessment, and the third meeting time Facilities and Management.



Conflict Graph

The origins of conflict graphs are in coloring maps, where regions which share a border must be different colors. However, the idea can easily be extended to cover other kinds of conflict as well, such as animals which cannot be placed in the same habitat, chemicals which cannot be stored in the same room, or school courses whose final examinations cannot be given at the same time. (Note: the zoo habitat problem appears on the video *Geometry,* available from COMAP.

An Election Followup Activity

by Sherida Hare

This is an activity that I used successfully in a precalculus class. I presented material and worked examples similar to those in [1] and [2] which took about 5 days. The presentation covered not only different voting methods, but also some of the paradoxes in voting.

After the students had a working knowledge of how voting works, we put together a questionnaire on the 1992 presidential election asking readers to rank the three candidates (Bush, Clinton, and Perot). We then looked at six vote-counting methods discussed in class: Majority-Rule, Plurality, Condorcet, Borda Count, Sequential, and Sequential Run-off [1, 2]. We polled the entire school and looked at the results of each method. Clinton won in all cases except for the 11th grade Borda Count in which Perot won. Bush came in last place in all of the results.

References

[1] COMAP, *For All Practical Purposes*, 3rd Ed., W.H. Freeman, New York, 1994, Chap. 11.

[2] L. Charles Biehl and Joseph G. Rosenstein,"Calling that *Mathematician...",* In Discrete Mathematics, No. 2 (October 1992), pp 11-12.